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ROUTE, AIRCRAFT PRIORITIZATION AND SELECTION FOR AIRLIFT MOBILITY OPTIMIZATION

by

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The *Throughput II* mobility optimization model (Morton, Rosenthal, and Lim, 1995) was developed at the Naval Postgraduate School for the Air Force Studies and Analysis Agency (AFSAA). The purpose of *Throughput II* is to help answer questions about the ability of the USAF to conduct airlift of soldiers and equipment in support of major military operations. Repeated runs of this model have helped AFSAA generate insights and recommendations concerning the selection of aircraft assets. Although *Throughput II* has earned the confidence of AFSAA, repeated applications are hampered by the fact that it can take over three hours to run on a fast workstation. This is due to the model's size; it is a linear program whose dimensions can exceed 100,000 variables, 100,000 constraints, and 1 million nonzero coefficients, even after extensive model reduction techniques are used. The purpose of this thesis is to develop heuristics that can be performed prior to running *Throughput II* in order to reduce the model's size. Specifically, this thesis addresses the fact that the *Throughput II* formulation has many variables and constraints that depend on the number of available routes for each aircraft. The goal is to carefully eliminate routes so as to make the problem smaller without sacrificing much solution quality.

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ROUTE, AIRCRAFT PRIORITIZATION AND SELECTION FOR AIRLIFT MOBILITY OPTIMIZATION

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ABSTRACT

The *Throughput II* mobility optimization model (Morton, Rosenthal, and Lim, 1995) was developed at the Naval Postgraduate School for the Air Force Studies and Analysis Agency (AFSAA). The purpose of *Throughput II* is to help answer questions about the ability of the USAF to conduct airlift of soldiers and equipment in support of major military operations. Repeated runs of this model have helped AFSAA generate insights and recommendations concerning the selection of aircraft assets. Although *Throughput II* has earned the confidence of AFSAA, repeated applications are hampered by the fact that it can take over three hours to run on a fast workstation. This is due to the model's size; it is a linear program whose dimensions can exceed 100,000 variables, 100,000 constraints, and 1 million nonzero coefficients, even after extensive model reduction techniques are used. The purpose of this thesis is to develop heuristics that can be performed prior to running *Throughput II* in order to reduce the model's size. Specifically, this thesis addresses the fact that the *Throughput II* formulation has many variables and constraints that depend on the number of available routes for each aircraft. The goal is to carefully eliminate routes so as to make the problem smaller without sacrificing much solution quality.

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EXECUTIVE SUMMARY

The magnitude of the airlift effort during Desert Shield/Storm was unprecedented. At the height of the war during Fall 1990, the Air Force averaged 17 million ston-miles per day of cargo and troops and by 10 March 1991, strategic airlift had moved more than 500,000 people and 540,000 stons of cargo (Gulf War Air Power Survey, 1993, p. 3). Due to the remarkable growth in the size and complexity of airlift operations, there is increased need for planning tools to assist decision makers.

The *Throughput II* mobility optimization model (Morton, Rosenthal, and Lim, 1995) was developed at the Naval Postgraduate School for the Air Force Studies and Analysis Agency (AFSAA). AFSAA asked NPS to develop *Throughput II* as a replacement for the first *Throughput*, which was developed by AFSAA (Yost, 1994). The model is formulated as a multi-period, multi-commodity linear programming model. It is implemented in the General Algebraic Modeling System (GAMS) (Brooke *et al.*, 1992). The purpose of *Throughput II* is to help answer questions about the ability of the Armed Forces of the United States to conduct airlifts of soldiers and equipment in support of major military operations. For a given fleet of aircraft, a given network of routes, and a given set of movement requirements over time, *Throughput II* schedules the airlift so as to minimize the penalty costs of late deliveries and non-deliveries.

Repeated runs of this model have helped AFSAA generate insights and recommendations concerning the selection of aircraft assets. It can also be used to investigate other strategic issues, such as investing or divesting in airfield infrastructure. Although *Throughput II* has earned the confidence of AFSAA, repeated applications are hampered by the fact that it can take over three hours to run on a fast workstation. This is due to the model's size.

The purpose of this thesis is to develop rapid computational methods that can be performed prior to running *Throughput II* in order to reduce the model's size. Specifically, this thesis addresses the fact that the *Throughput II* formulation has many variables and constraints that depend on the number of available routes for each aircraft. The goal is to carefully eliminate routes so as to make the problem smaller without sacrificing much solution quality.

We define an efficiency factor for each route-aircraft type pair and use it to compare the routes. The efficiency factor is a simple ratio based on aircraft and route properties, and the cargo to be moved on the given route.

The characteristics of route-aircraft pairs used to define the route efficiency factor are:

- Cargo carriage capacities
- Maximum flight distance for a given payload
- Onload, offload, and ground times at enroute airfields
- Aircraft block speed

The route efficiency factor is defined differently for delivery and recovery routes. In both cases it attempts to unite all the above properties into a single numerical value which we can use to prioritize route-aircraft pairs.

Simply ranking the aircraft/route pairs by route efficiency factor is not a rigorous enough screening of possibilities for *Throughput II*. The primary reason is that it ignores constraints on airfield capacity, known as MOG (Maximum on Ground). The maximum number of aircraft that can be sent through an airfield depends on many dimensions such as: number of parking spaces at an airfield, material handling equipment, ground services capacity and fuel availability.

We formulate and solve aggregated, simplified versions of *Throughput II* as a method of screening aircraft/route pairs. For example, we may attempt to move several days worth of cargo and passengers in a single day. These versions put considerable stress on the mobility system, particularly MOG constraints. We observe which routes are used

in the solutions of these simple models and let them become candidate routes to be used by *Throughput II*. By varying the conditions under which the simple models are defined, a very good, but not overly large, set of candidate routes is generated for *Throughput II*.

The procedures introduced in the thesis were tested on a typical mobility problem. Route-aircraft pairs were reduced approximately 52%. We observed 75% improvement on the runtime of the *Throughput II*. However, the optimum value of the *Throughput II* solution went up 2.4%.

When the size of the original model is too large for contemporary solvers, route selection may be even more critical. A mobility network structure with fewer routes is also more realistic from an operations standpoint.

I. INTRODUCTION

In addition to providing combat forces necessary to prevent or fight a general war, the Air Force is responsible for providing airlift for use by all other military services. Prior to the 1930's, the deployment and logistical support of military forces was undertaken mostly via surface transport systems. There are three fundamental disadvantages of surface deployment, which cannot be eliminated by speed and efficiency provided by the newly developed ideas and technology of this century. First, surface deployment over substantial distances is slower than air transport systems. Secondly, surface deployment is heavily restricted by geographical constraints. The third, and most important reason, concerns its susceptibility to enemy attacks. On the other hand, airlift has less capacity than the sealift does, but because of its speed and range, all modern armed forces have incorporated airlift into their transportation systems.

The U.S. moved 1.7 million ton-miles per day during the Berlin Airlift, and 4.4 million ton-miles per day during the 1973 Arab-Israeli War. As an example to the most recent airlift operations, during Desert Shield and Desert Storm, strategic airlift had moved over 500,000 people and 540,000 tons of cargo, with cargo movement averaging 17 million ton-miles per day (Gulf War Air Power Survey, 1993).

Airlift aircraft provide the capability to deploy armed forces anywhere in the world and help sustain them in a conflict. Airlift aircraft consist of military and the Civil Reserve Air Fleet (CRAF). Selected civil aircraft from U.S. airlines, contractually committed to CRAF, support Department of Defense airlift requirements in emergencies when the airlift need exceeds the capability of military aircraft.

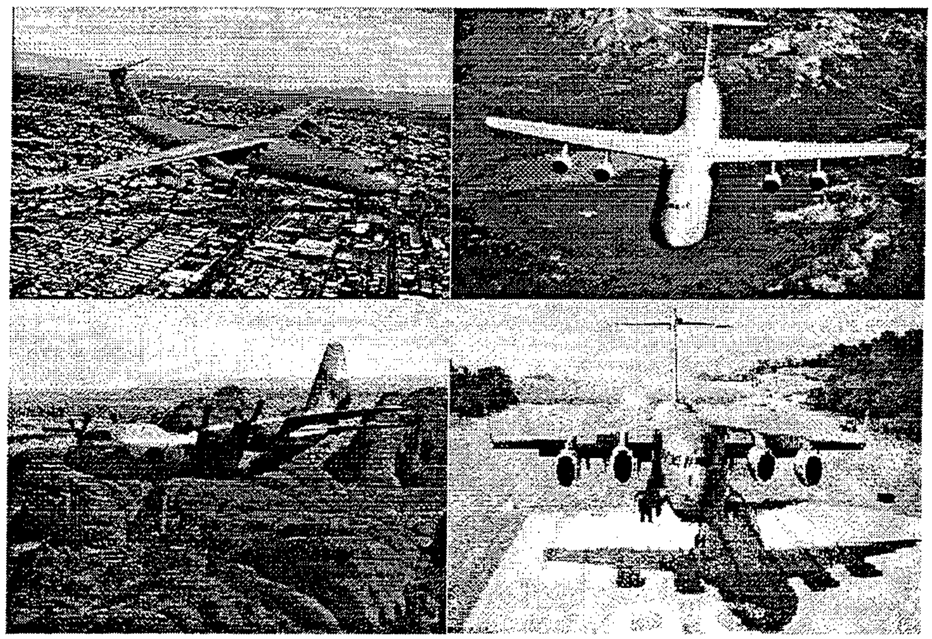

Figure 1. *During Desert Shield and Desert Storm Operations, the United States Air Force had moved over 500,000 people and 540,000 tons of cargo, with cargo movement average 17 million ton-mile per day. Air assets used in Airlift operations consist of military and civil reserve aircraft. Some military aircraft types used in Airlift operations are shown above (C-141; C-5; C-130; C-17).*

According to the Gulf War Air Power Survey:

> The primary goal of strategic airlift planning is to satisfy the Commander-in-Chief, Central Command's (CINCCENT) requirements in contingencies by employing airlift resources effectively. To meet the goal, planners have to consider the entire airlift system and its interrelated parts (Gulf War Air Power Survey, 1993).

Aircraft have to arrive where and when they are needed. Each stop along the way has to have adequate runways, taxiways, ramps, and support facilities. Nonproductive ground time has to be minimized. The necessary equipment and trained personnel have to be on hand to load and unload passengers and cargo.

Due to the remarkable growth in the size and complexity of airlift operations, there is increased need for planning tools to assist decision makers. To provide the assistance to the decision maker, Operations Research techniques that search for the optimum solution to complex airlift problems can be used. Although it must be interpreted carefully, this "search for optimality" is a very important theme in Operations Research. The field of Operations Research provides tools for attempting to find the best, or optimal, solution to mathematical models of the problem under consideration.

A. BACKGROUND

The *Throughput II* mobility optimization model (Morton, Rosenthal and Lim, 1995) was developed at the Naval Postgraduate School for the Air Force Studies and Analysis Agency (AFSAA). AFSAA sponsored research began at NPS in late 1993 and continues through the present time. AFSAA asked NPS to develop *Throughput II* as a replacement for the first *Throughput*, which was developed by AFSAA (Yost, 1994).

The purpose of *Throughput II* is to help answer questions about the ability of the Armed Forces of the United States to conduct airlift of soldiers and equipment in support of major military operations. For a given fleet of aircraft, a given network of routes, and a given set of movement requirements over time, *Throughput II* schedules the airlift so as to minimize the penalty costs of late and non-deliveries. Repeated runs of this model have helped AFSAA generate insights and recommendations concerning the selection of aircraft assets. It can also be used to investigate other strategic issues, such as investing or divesting in airfield infrastructure.

B. PROBLEM STATEMENT

Although *Throughput II* has earned the confidence of AFSAA, repeated applications are hampered by the fact that it can take over three hours to run on a fast

workstation. This is due to the model's size; it is a linear program whose dimensions can exceed 100,000 variables, 100,000 constraints, and 1 million nonzero coefficients, even after extensive model reduction techniques are used.

The purpose of this thesis is to develop heuristics that can be performed prior to running *Throughput II* in order to reduce the model's size. Specifically, this thesis addresses the fact that the *Throughput II* formulation has many variables and constraints that depend on the number of available routes for each aircraft. The goal of this research is to carefully eliminate routes so as to make the problem smaller without sacrificing much solution quality.

In addition to the computational benefits to be gained from this thesis, the simplified route structure that results from our work may be a side benefit. In reality, it is difficult and expensive to maintain bases all around the world. Results obtained from this thesis work may help decision makers investigate which routes, and consequently which airfields have high benefit in terms of air mobility.

C. METHODOLOGY

1. Pre-prioritization of Aircraft/Route Pairs

A criterion is needed in order to be able to make comparisons and selections between available routes. We define an efficiency factor for each route-aircraft type pair and use it to compare the routes. The efficiency factor is a simple ratio based on aircraft and route properties.

The characteristics of route-aircraft pairs used to define the route efficiency factor are:

- Cargo carriage capacities
- Maximum flight distance for a given payload
- Onload, offload, and ground times at enroute airfields
- Aircraft block speed

The route efficiency factor is defined differently for delivery and recovery routes. In both cases it attempts to unite all the above properties into a single numerical value which we can use to prioritize route-aircraft pairs.

Simply ranking the aircraft/route pairs by route efficiency factor is not a rigorous enough screening of the possibilities for *Throughput II.* The primary reason is that it ignores constraints on airfield capacity, known as MOG (Maximum on Ground). The maximum number of aircraft that can be sent through an airfield depends on many factors such as: number of parking spaces at an airfield, material handling equipment, ground services capacity and fuel availability. We need another method which incorporates this aspect of the airlift system in order to avoid airfield congestion in the construction of the prioritized list of route-aircraft pairs. We will develop simple mathematical models and use them as heuristics for this purpose.

2. Generating Candidate Aircraft/Route Pairs with Fast Linear Programs

We formulate and solve aggregated, simplified versions of *Throughput II* as a method of screening aircraft/route pairs. For example, we may attempt to move several days worth of cargo and passengers in a single day. This will put considerable stress on the mobility system, particularly with respect to MOG constraints. We can observe which routes are used in the solutions of these simple models and let them become candidate routes to be used by *Throughput II.* By varying the conditions under which the simple models are defined, a very good, but not overly large, set of candidate routes can be generated for *Throughput II.*

Air mobility problems may become very complex. For example, conflicts may occur in more than one place at the same time. In this thesis we will also develop extensions of the above ideas for more complex situations. A sample air mobility problem will be used to test the developed procedures in those possible complex cases.

D. SUMMARY

Mobility modeling for the USAF by optimization necessarily involves solving large problems. In order to reduce the size of a mobility linear program, route-aircraft pairs may be preprocessed to select only the most promising candidates. The methodology presented in this thesis reduces the number of routing decisions significantly with only a small loss of optimality. On the other hand, reduction in the size of inputs provided by the methods described improves the runtime of mobility models such as *Throughput II*.

II. REVIEW OF THROUGHPUT II MODEL

The *Throughput II* mobility optimization model (Morton, Rosenthal, and Lim, 1995) was developed and enhanced at the Naval Postgraduate School. The mathematical formulation of *Throughput II* is included in Appendix A. The objective of the *Throughput II* model is to determine the maximum on-time delivery of cargo and passengers that can be transported with a given fleet of aircraft over a given network. *Throughput II* schedules the airlift so as to minimize the penalty costs of late and non-deliveries.

The purpose of this thesis is to develop a heuristic that can be performed prior to running *Throughput II* in order to reduce the model's size. A clear understanding of the *Throughput II* model and its size is required in order to motivate this thesis, so we provide a review of *Throughput II* in this chapter.

A. MODEL FEATURES

The *Throughput II* model has been designed to handle many of the airlift system's particular features and modes of operation. The model is a strategic airlift model, meaning that it considers inter-theater but not intra-theater deliveries. The major features of the

airlift system currently captured by the model include (Morton, Rosenthal, and Lim, 1995):

- Multiple origins and destinations: The model routes aircraft through multiple origin, enroute and destination airfields.

- Flexible routing structure: The air route structure supported by the model includes delivery and recovery routes with a variable number of enroute stops (usually between zero and three). This gives the model the option of short-range flights with heavier loads or long-range flights with lighter loads. For further routing flexibility, the model also allows the same aircraft to fly different delivery and recovery routes on the same mission.

- Aircraft-to-route restrictions: The user may impose aircraft-to-route restrictions; e.g., military aircraft may only use military airfields for enroute stops. This particular provision arises because the USAF Air Mobility Command (AMC) may call upon civilian commercial airliners to augment USAF aircraft in a deployment, under the Civil Reserve Airlift Fleet (CRAF) program. The model distinguishes between USAF and CRAF aircraft.

- Aircraft assets can be added over time. This adds realism to the model because CRAF and other aircraft may take time to mobilize and are typically unavailable at the start of a deployment.

- Delivery time windows: In a deployment, a unit is ready to move on its available-to-load date (ALD) and has to arrive at the theater by its required-delivery-date (RDD). This aspect of the problem has been incorporated in the model through user-specified time windows for each unit. The model treats this time window as "elastic" in that cargo may be delivered late, subject to a penalty.

B. ASSUMPTIONS (Morton, Rosenthal, and Lim, 1995)

The assumptions used in the model are as follows:

- Airfield capacity is represented by a single aggregate figure, called *Maximum-on-Ground* (MOG). The literal translation of MOG as the maximum number of planes that can be simultaneously on the ground at an airfield is misleading, because MOG is used to convey more than just the number of parking spaces. In actuality, airfield capacity depends on many factors such as availability of

material handling equipment and various ground servicing capacities. Unfortunately, data are not currently available to support a multidimensional MOG modeling enhancement.

- Inventoried aircraft at origin and destination airfields are considered not to affect the aircraft handling capacity of the airfield. This assumption is not strictly valid since an inventoried aircraft takes up parking space even if is not consuming services.

- Deterministic ground time: Aircraft turnaround times for onloading and offloading cargo and enroute refueling are assumed to be known constants, although they are naturally stochastic. This ignores the fact that deviations from the given service time can cause congestion on the ground. To offset the optimism of this assumption, an efficiency factor is used in the formulation of airfield capacity constraints.

C. CONCEPTUAL MODEL FORMULATION (Morton, Rosenthal, and Lim, 1995)

The primary decision variables are the number of sorties initiated, and the amount of cargo and passengers carried, for each unit, by each aircraft type, via each available

route, in each time period. Additional variables are defined for the recovery flights, for aircraft inventoried at airfields, and for the possibility (at high penalty cost) of not delivering required cargo or passengers.

1. Objective Function

The objective function minimizes the total weighted penalties for late deliveries and nondeliveries subject to appropriate physical and policy constraints. The penalties are weighted according to two factors: the priority of the unit and the degree of lateness. The penalty increases with the amount of time late, thus non-delivery has the most severe penalty.

The model's anticipated use regards situations when the given airlift resources are insufficient for making all the required deliveries on time. On the other hand, if there are enough resources for complete on-time delivery, then the model's secondary objective function is to choose a feasible solution that maximizes unused aircraft. The motivation for the secondary objective is that if the available aircraft are used as frugally as possible, while still meeting the known demands and observing the known constraints, then the mobility system will be as well prepared as possible for unplanned breakdowns and unforeseen requirements, such as an additional contingency.

2. Constraints

The model's constraints can be grouped into five categories: demand satisfaction, aircraft balance, aircraft capacity, aircraft utilization, and airfield handling capacity.

- Demand Satisfaction Constraints: The cargo demand constraints attempt to ensure for each unit that the correct amount of cargo moves to the required destination within the specified time window. The passenger demand constraints do the same for each unit's personnel. The demand constraints have elastic variables for late delivery and non-delivery. The optimization will seek to avoid these options if it is possible with the available assets, or to minimize them if not.

- Aircraft Balance Constraints: These constraints keep physical count of aircraft by type (e.g., C17, C5, C141, etc.) in each time period. They ensure that the aircraft assets are used only when they are available.

- Aircraft Capacity Constraints: There are three different kinds of constraints on the physical limitations of aircraft – troop carriage capacity, maximum payload, and cabin floor space – which must be observed at all times.

- Aircraft Utilization Constraints: These constraints ensure that the average flying hours consumed per aircraft per day are within AMC's established utilization rates for each aircraft type.

- Aircraft Handling Capacity at Airfields: These constraints ensure that the number of aircraft routed through each airfield each day is within the airfield's handling capacity.

D. **LIMITATIONS** (Morton, Rosenthal, and Lim, 1995)

1. Time Resolution

Although the aircraft handling capacity of each airfield is observed by the model, airlift missions may still be routed in a manner that causes local congestion. For example, all aircraft routed through an airfield on a particular day could arrive within a small time window instead of being spread over the whole day. In reality, this would cause local congestion, even though the model representation aircraft handling capacity is observed.

2. Aircraft Reliability

Aircraft in need of repair can have an immediate impact on throughput capacity, especially when airfield capacity resources are limited.

3. Deterministic Ground Time

The aircraft ground times used in the calculation of MOG consumption represent the expected times for onload/refuel/offload, resulting in optimistic throughput capacity.

4. Airfield Aggregation

Throughput II uses data aggregated from the TPFDD for input. It replaces the large set of airfields with a smaller set of aggregated airfields and schedules aircraft through these aggregate airfields.

III. PRIORITIZATION OF ROUTE-AIRCRAFT PAIRS

Available routes for each aircraft type are one of the inputs to an air mobility model. Although optimality can be preserved by including all possible routes for each aircraft type, the solution time suffers badly due to the excessive dimensionality of the variable space. In the previous chapter we reviewed the *Throughput II* model. Consider a typical run, where there are two destination and four origin airfields (eight Origin-Destination pairs), and each unit has a 24 period delivery "window". Since all the units and routes have different origin destination pairs we can calculate the approximate numbers of *units per origin-destination pair*, and *routes per origin destination pair* by dividing the total number of units and the total number of routes by the number of origin-destination pairs. The decision variables of the model might have the following indices and dimensions:

u (units) : **228** $\Rightarrow$ **29** units/origin-destination pair (228/8)

a (aircraft) : **7**

r (routes) : **110** $\Rightarrow$ **14** routes/origin-destination pair (110/8)

t (time) : **24** average time

These numbers imply that for each route, the approximate number of variables generated will be as many as $29 \times 7 \times 24$. Thus, every route eliminated will cause 4872 potential column reductions. Our concern in this thesis is the number of route-aircraft pairs we supply to *Throughput II*. To be able to eliminate some of the route-aircraft pairs, a comparison and criterion to compare these pairs is required. In this chapter we will address to the properties of aircraft and routes that may help us to do the comparison. We will develop formulas to represent route-aircraft properties and capacities.

In a mobility problem, aircraft in the system have different characteristics. One would tend to favor aircraft with high capacity and long range, assuming near equivalence of other characteristics. Usually the distances between origins and destinations are long enough to require the use of an enroute airfield on the way. The cargo carriage capacity of an aircraft changes with the distance it flies; as the amount of cargo carried increases, the range is decreased, and the plane has to make more stops.

Figure 2. Depicts how the carriage capacity of an aircraft varies with the distance flown, for a set of aircraft types that can be used in a mobility system. This relationship is called the *range-payload curve*.

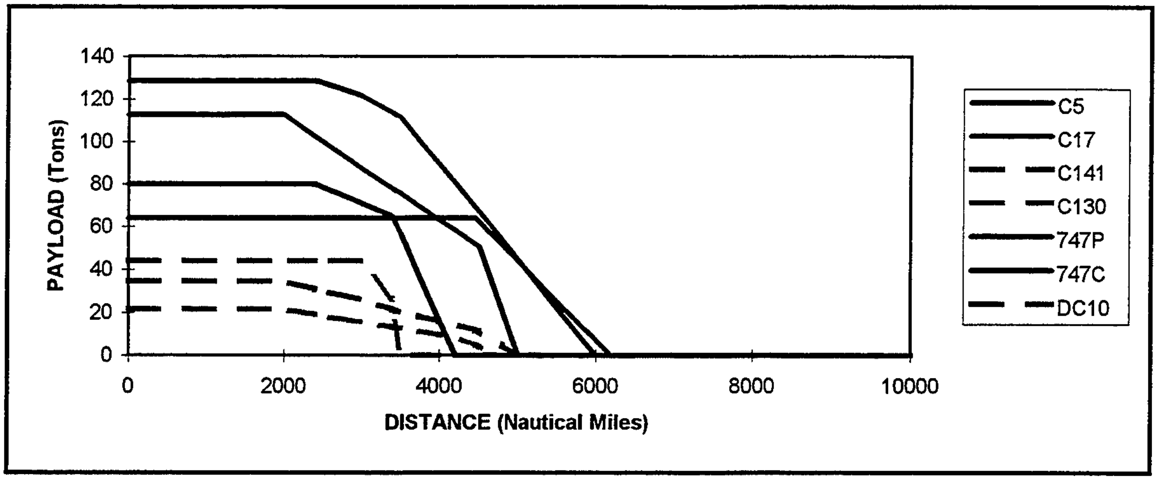

Figure 2. *Range-Payload curves: The change in the carriage capacity of aircraft types affects the distance they can fly. The horizontal axis is the distances in nautical miles, the vertical axis is the cargo amount in tons. For example, an aircraft of type C-5 can fly up to 3000 miles with 90 tons of cargo.*

Since every aircraft type has different characteristics, and the cargo capacities are dependent on the routes flown, some route-aircraft pairs are more effective than others. Relatively direct routes with evenly placed stops are preferred. If we can define a numerical value to represent these route characteristics, we can rank order the available aircraft-route pairs. If we can also show that route-aircraft pairs picked by a mobility optimization program like *Throughput II* and the calculated numeric values are related, then the numeric values will be beneficial to the analyst.

A. DERIVATION OF THE ROUTE EFFICIENCY FACTOR

To prioritize route-aircraft pairs, one needs to consider characteristic differences between aircraft types and routes. Properties of aircraft that need to be considered are as follows:

- Cargo carriage capacities
- Flight distance for a given payload
- Onload / offload times, and ground times at enroute airfields
- Aircraft block speed
- Aircraft handling capacity consumption at each airfield
- Allowable enroute airfields

In order to incorporate the above characteristics, we need to define and evaluate a criterion which unites these properties for all route-aircraft pairs. With such *a Route Efficiency Factor*, we can order all route-aircraft pairs.

If the route efficiency factor is accurate, we will expect to see routes that price favorably in the linear program to have a high Route Efficiency Factor. Factors for recovery and delivery routes must be calculated differently, but the underlying efficiency should be similar.

B. DELIVERY ROUTES

Delivery routes are the routes that aircraft fly to carry cargo and personnel from the embarkation airfields to the debarkation airfields. For example, in an operation like Desert Storm, armed forces will be transported from Travis in CONUS (Continental United States) to Dhahran, Saudi Arabia, so Travis - Dover - Mildenhal - Dhahran may be a delivery route. Because of the range payload trade off, aircraft payload will be limited by the flight distances of the legs along this route. Additionally, stops for crew replacement will be required. The longest distance that an aircraft flies on a given route is called the *Critical Leg* of the aircraft on the route. An example is constructed below to explain *Critical Leg*:

Let's assume that a given route from origin airfield (APOE) to destination airfield (APOD) has two enroute airfields, and distances between airfields and corresponding payloads for a C-5 type of aircraft are:

APOE - ENROUTE1 = 1500 MILES / 1127 TONS

ENROUTE1 - ENROUTE2 = 3500 MILES / 757 TONS

ENROUTE2 - APOD = 1000 MILES / 1127 TONS

The critical leg of this route for a C-5 is 3500 Miles, which limits the payload for the entire route.

An implicit objective of the mobility system is to carry cargo as early as possible within the required delivery window of each unit. This implies the importance of aircraft speed, total distance of the route, and total time an aircraft spends on the ground. A numerical measure that incorporates all of these factors can be used to make a choice among the many route-aircraft pairs. First we will define the following terms which correspond to the characteristics listed above.

$\boldsymbol{P_{ar}}$: Maximum payload for aircraft type $\boldsymbol{a}$ on route $\boldsymbol{r}$ depending on the critical leg of the route;

$\boldsymbol{D_r}$: Total distance from origin to destination on route $\boldsymbol{r}$;

$\boldsymbol{S_a}$: Speed of aircraft type $\boldsymbol{a}$;

$\boldsymbol{G_{ar}}$: Total time spend on the ground on route $\boldsymbol{r}$ by aircraft type $\boldsymbol{a}$;

$\boldsymbol{EF_{ar}}$: Efficiency Factor of $\boldsymbol{a}$-$\boldsymbol{r}$ aircraft-route pair;

The proposed Efficiency Factor is:

$$\boxed{EF_{ar} = \frac{S_a \cdot P_{ar}}{D_r + S_a \cdot G_{ar}}}$$

The units of the Efficiency Factor formulated above are $\frac{Tons}{Hour}$. Since the objective of the air mobility problem is to minimize undelivered cargo, routes that carry more cargo and take less time will be preferred. An aircraft with a larger cargo carrying capacity on a shorter route will probably deliver more cargo throughout the scenario.

Confounding this calculation is the fact that capacity of an aircraft is also dictated by cargo volume. There may be some kind of cargo which is lighter but very large. The density of each unit's cargo is given in square feet per ton. In addition to the previous notations we will use following terms in our calculations:

U_i	Subset of units whose origin airfield is ***i***;
i_r	Origin airfield of route ***r;***
$Weight_u$	Weight of unit ***u*** in tons;
$FloorAreaPerTon_u$	Floor area per ton that unit ***u*** covers;
$AvgFloorAreaPerTon_i$	Average floor area per ton covered by the units to be moved from origin airfield ***i***;
$AreaCapacity_a$	Area capacity of aircraft type ***a*** in square feet;

The average density of the units at origin airfield i is calculated below:

$$\frac{\sum_{u \in U_i} Weight_u \cdot FloorAreaPerTon_u}{\sum_{u \in U_i} Weight_u} \cong AvgFloorAreaPerTon_i$$

This approximation of density will be used to check if the cargo loaded in an aircraft is within its volume limits. If it is not the $\boldsymbol{P_{ar}}$ (Payload) will be modified using the average density at origin of route $\boldsymbol{r}$.

$$\textbf{\textit{if}} \qquad P_{ar} \cdot AvgFloorAreaPerTon_{i_r} \geq AreaCapacity_a \quad \textbf{\textit{then}}$$

$$P_{ar} = \frac{AreaCapacity_a}{AvgFloorAreaPerTon_{i_r}} \; ;$$

This adjusted figure for payload will be used in the Route Efficiency Factor calculations.

C. RECOVERY ROUTES

Recovery routes are those routes that aircraft fly returning to an embarkation airfield after making a delivery. For the example described above, Dhahran - Mildenhal - Dover - Travis will be a recovery route. Aircraft will be empty when they are flying back, so they can fly longer distances. We want to do recovery quickly, because as soon as we recover an aircraft we can prepare and send it again. This will shorten the closure time of the mobilization.

Characteristics and properties we need to consider for recovery routes will be somewhat different from considerations for delivery routes. Speed, distance and ground time will still be factors, but aircraft on recovery routes are empty and can fly at their maximum range. Thus, Efficiency Factors for Recovery routes will be calculated with a different formula.

Definitions of D_r, S_a, G_{ar} are same as delivery routes.

Efficiency factors for recovery routes are defined as:

$$\boxed{EF_{ar} = \frac{S_a}{D_r + S_a \cdot G_{ar}}}$$

The units of the Efficiency Factor for recovery routes are $\frac{1}{Hour}$. Since aircraft will be empty on the recovery routes, payload is irrelevant and the main concern is the distance they fly. Thus, recovery routes that allow aircraft to get back to origin airfields in the shortest time will have higher priority. To maintain consistency between delivery and recovery routes, the efficiency factor for recovery routes is defined as speed over total distance of the route, including ground times.

Routes will be sorted from best to worst using the Efficiency Factors defined above, for each origin-destination pair and type of aircraft. The next step is to decide how many of these routes to include in *Throughput II*.

D. VERIFICATION OF ROUTE EFFICIENCY FACTORS

We need to verify correctness of the tool we developed. For this verification, we used the mobility scenario data set in (Lim, 1994). We ran a typical scenario on *Throughput II* and recorded the mission numbers flown on each of the route-aircraft pairs. Then we calculated the "route efficiency factor" for each of these pairs. We picked an origin-destination pair arbitrarily to observe the relation between efficiency factors and missions flown on the routes for this pair. For each aircraft type we plotted the number of

missions flown vs. "route efficiency factor" calculated. We observed that all the route-aircraft pairs with a high number of missions flown have high efficiency factors. Graphical results of this work are shown below. We indicate suggested cut points with a dashed line on the graphs. These cut point values of efficiency factors result from visual investigation of the graphs.

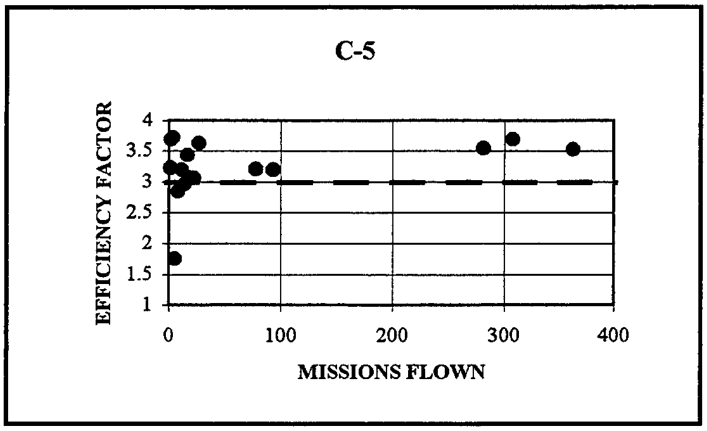

Figure 3 *Missions Flown vs. Efficiency Factors for C-5 routes. For aircraft type C-5, routes that are used more have a high "Route Efficiency Factor". For example, routes that have 281, 307, and 361 missions throughout the scenario, have efficiency factors of more than 3.5. We expect that deleting the route-aircraft pairs below efficiency level 3 does not change the result of the problem much since they have few missions flown.*

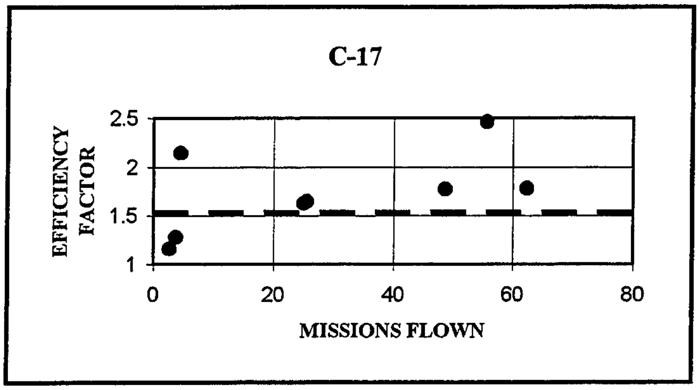

Figure 4 *Missions Flown vs. Efficiency Factors for C-17 routes. Eliminating the routes with efficiency factors 1.2 and 1.3 is expected not to affect the solution much, since the number of missions flown on these routes is small.*

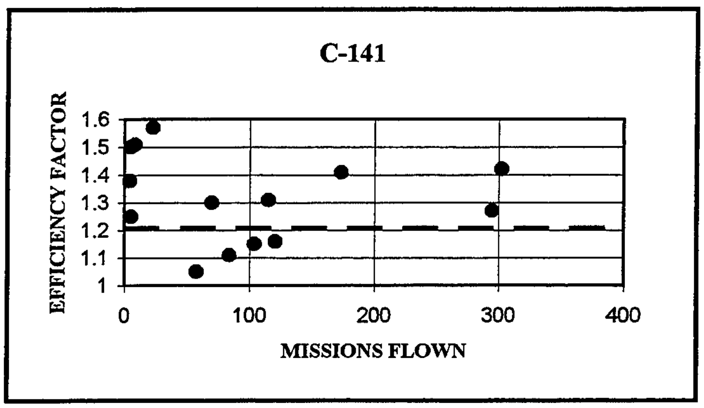

Figure 5 *Missions Flown vs. Efficiency Factors for C-141 routes. Results of this graph seems in contradiction with the others, however routes with higher efficiency and lower missions flown may be saturated by other types of aircraft. In these graphs we investigate aircraft types independently, but mobility model does not handle aircraft separately. Some of the routes may be more efficient for other aircraft. Thus, routes utilized by other aircraft may not be available for C-141. We observe that route-aircraft pairs with efficiency factor less than 1.2 are ranked fourth or lower in number of missions flown. Routes that are flown more have higher efficiency factors.*

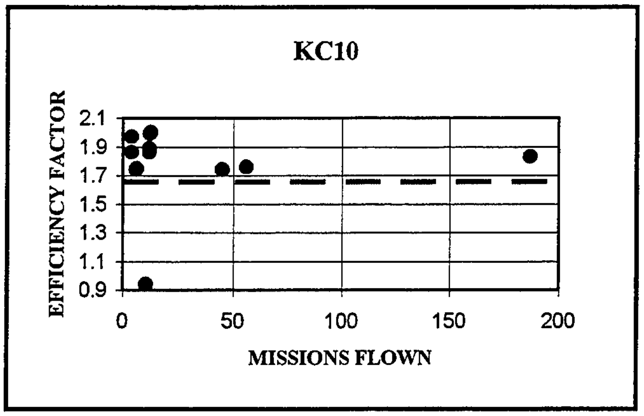

Figure 6 *Missions Flown vs. Efficiency Factors forKC-10 routes. None of the routes that have the smallest number of missions have high route efficiency factors. We may remove the route-aircraft pair that has efficiency factor less than 1.7.*

As depicted by the Figures 3 to 6, routes that are flown more (for C-5 more than 80, for C-17 more than 20, for C-141 more than 150, for KC-10 more than 40) have high route efficiency factor values. None of the route-aircraft pairs that has smallest efficiency factor have been flown a lot (for C-5 and C-17 less than 5, for KC-10 less than 10) with the exception of C-141. This implies that we can use route efficiency factor to compare route-aircraft pairs and to decide whether we need to keep a route-aircraft pair in the model. For some aircraft, a few routes with low efficiency factors get used. The reason for this may be the other constraining factors in the mobility model, such as aircraft handling capacities of airfields. If the route-aircraft pairs with higher efficiency factor saturate some airfields, less efficient aircraft may fly on routes with lower efficiency.

E. CONCLUSION

Among all the available route-aircraft pairs for a mobility problem, some will be used more than others, because of the different characteristics of these route-aircraft pairs. We developed a simple criterion in order to compare route-aircraft pairs. This comparison may help us eliminate some of the less efficient route-aircraft pairs. This issue will be addressed in the next chapter.

IV. MATHEMATICAL MODEL FOR ROUTE ELIMINATION

Although we have defined a criterion to compare route-aircraft pairs in the previous chapter, we still do not know how many of these routes we can eliminate without adversely affecting the mobility model. We intend to use linear programming models to be able to decide which of these routes we need to use in *Throughput II*. Using the efficiency factor as an objective to be maximized, we formulate and solve aggregated, simplified versions of *Throughput II* as a method of screening aircraft-route pairs. We observe which routes are used in the solutions of these simple models and let them become candidate routes to be used by *Throughput II*. The models introduced in this chapter are too simplified to be used on the mobility problem itself. The idea of the new model is to put large, perhaps unrealistic, movement demands on the mobility system, and then see which routes are used under these stressful conditions.

The route structure in an air mobility problem can be represented by a network diagram. In Figure 7, an example for the route structure is shown.

In an air mobility problem, the number of aircraft available, the total amount of cargo to be moved, and airfield capacities are the main concerns. The throughput amount is partially dependent on aircraft handling capacities of airfields. At this point, the

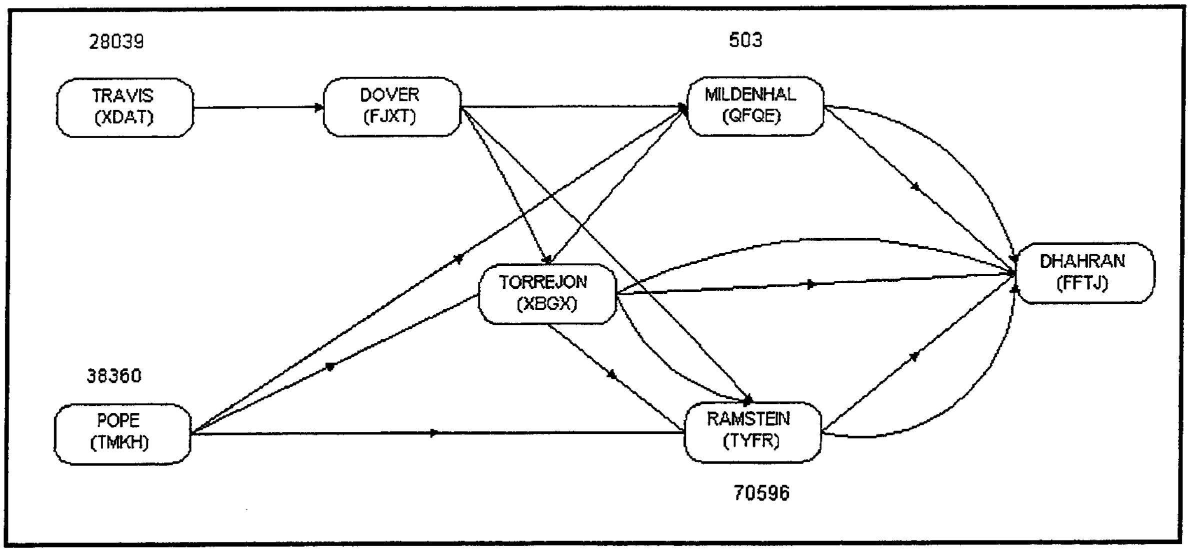

Figure 7 *In the air mobility network diagram there are four origin airfields : Travis, Pope, Mildenhal, and Ramstein. The destination airfield is Dhahran. Dover and Torrejon are enroute airfields. Also Mildenhal and Ramstein are enroute airfields for aircraft coming from either Travis or Pope. The goal in this problem is to carry as much cargo as possible from origin airfields to the destination airfield in a given amount of time. The cumulative cargo amounts at origin airfields are shown in the figure. The total number of aircraft missions flown is constrained by the aircraft handling capacities of airfields.*

definition of MOG (Maximum on Ground) becomes important. Basically MOG represents the capacity of an airfield. Airfield capacity depends on many dimensions such as:

1. Number of parking spaces at an airfield;
2. Material handling equipment;
3. Ground services capacity;
4. Fuel availability.

The maximum number of aircraft that can be sent simultaneously through an airfield is limited by the MOG capacity of that airfield. Available MOG at airfields should be incorporated as a constraint in the models developed in this chapter, in order to discourage selection of routes which use MOG inefficiently relative to others.

A. DELIVERY ROUTES

We will construct a cargo delivery model maximizing the sum of the efficiency factors of the selected routes, constrained by the number of aircraft available, total cargo available, and MOG available at each airfield.

Airfields have different capacities depending on the aircraft type. To use our mathematical models, we will define a parameter to represent the MOG consumption percentage at a given airfield for a given aircraft type. Using aircraft speed and route leg lengths we can also specify when an aircraft launched at a given time period will consume MOG. We will call this parameter "CONSUMEP." This parameter will be defined for all aircraft types, airfields, routes and time periods.

Since the combination of cargo amount and aircraft fleet size/structure may yield different solutions in the model we are going to construct, and cargo becomes available at origins in different times and in different amounts, investigating the given scenario during

different intervals is appropriate. Logical investigation points are the days just before new aircraft are allocated to the fleet. Aspects of the problem change at these times, thus dividing the problem into subproblems defined by allocation times is appropriate. At these investigation points, the cargo amount will be the cumulative cargo at origins, and number of aircraft will be the number just prior to allocation.

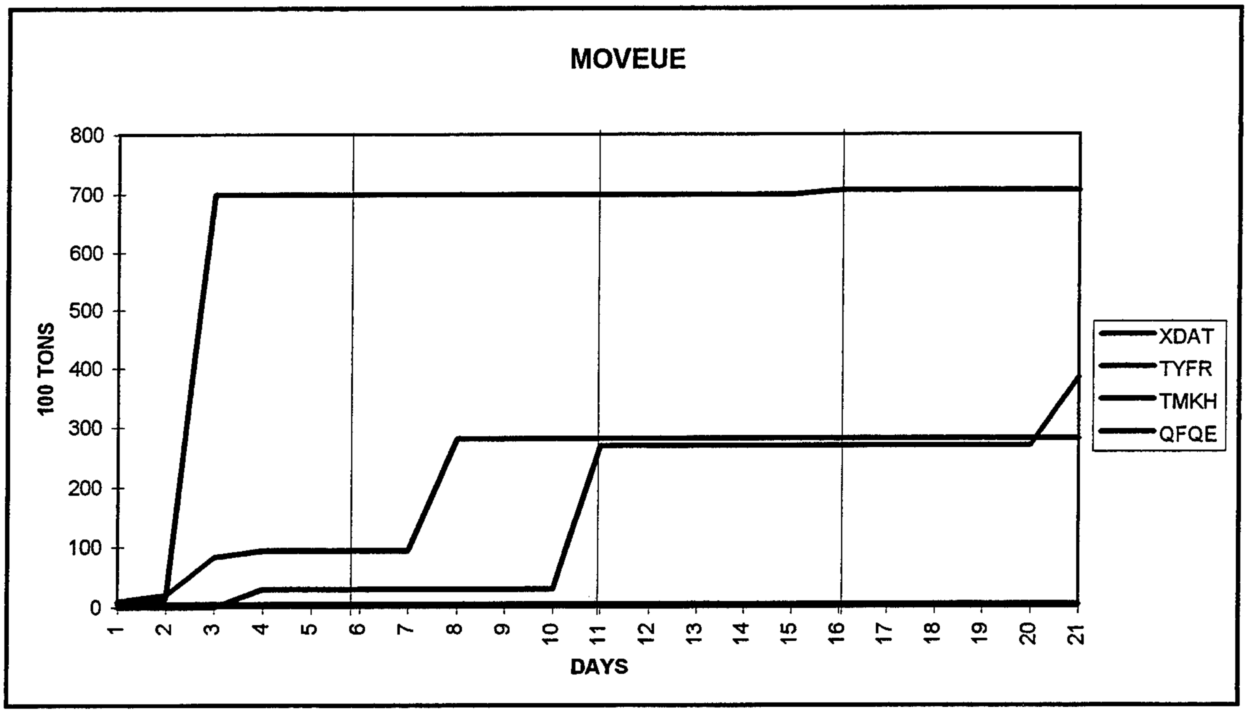

Figure 8 *This figure depicts cumulative amount of cargo at origin airfields(XDAT, TYFR, TMKH, QFQE). The vertical lines are new aircraft supply dates (1, 6, 11, 16).*

For the example shown in Figure 8, the scenario will be divided into four intervals:

I. DAY1 - DAY6;

II. DAY7 - DAY11;

III. DAY12- DAY16, and,

IV. DAY17 - DAY21.

To stress the network we will create extreme conditions in cargo amounts. Within each of these intervals the available cargo on the last day of the interval will be used as the total cargo available to move at origin airfields, and aircraft fleet structure will be the total number of aircraft in first day of the intervals. The linear model will be run for each of these intervals, and aircraft-route pairs that are selected to carry cargo will be retained. These intervals will be independent of each other. The intervals will differ due to the number of aircraft available, and cargo amount at the origin airfields. Within each of these intervals we will launch as many aircraft as the network allows, and we will call each of these aircraft launch times a *pass*. The constraining factors on passes and calculation of times between each pass will be described in the following parts of this chapter.

Since we use some assumptions to stress the network, we need to make sure that all possible conditions are covered by our assumptions. We will achieve this by running our model for every relevant combination of origin-destination pairs.

Assumptions and simplifications for stressing the network are as follows:

1. We will assume that cargo amount at an airfield is the cumulative amount up to the time of interest.
2. For the delivery model we will assume that we don't replenish aircraft within an interval.
3. We will run our models for:
   - each origin-destination pair separately
   - all origin-destination pairs in each major contingency region (if more than one exist)
   - all major contingencies simultaneously

We don't include the demand of cargo at destination airfields in our simplified models. Consequently, the runs involving multiple origin-destination pairs may not select routes for all these pairs, but will be supplemented by routes chosen in the single origin-destination pair runs. These single origin-destination pair runs provide solutions for extreme cases which compensate for the loss of not including the demand.

Indices and parameters that we will use in our formulations are as follows:

INDICES:

dr : Delivery routes;

a : Aircraft type (i.e. C-5, C-17, etc.);

af : Airfields in general;

i : Origin airfields (APOE's);

e : Enroute airfields ;

k : destination airfields;

p : pass start times;

t : time;

intv : intervals of time that we investigate the mobility problem;

PARAMETERS:

$EF_{a,dr}$ Route Efficiency Factor of *dr,a* route-aircraft pair;

$CONSUMEP_{a,af,dr,t}$ Percentage of MOG consumed at airfield *af* at time *t* by single a/c of type *a* when it flies on route *dr*;

$PAYLOAD_{a,dr}$ Amount of payload that a single a/c of type *a* can carry on route *dr*;

$TOTMOVE_i^{intv}$ Total tons of cargo to be moved from origin *i* during interval *intv*;

$ACAVAIL_a^{intv}$ Number of a/c of type *a* available on interval *intv*;

C_p	Constant that motivates early deliveries, value of C is bigger for small p's.

DECISION VARIABLE :

$X_{a,dr,p}^{intv}$	Number of a/c of type *a* sent on route *dr* on pass *p* on interval *intv*;

Explanations for the objective function and constraints of the linear programming model are as follows:

1. Objective Function

$$\text{Maximize} \qquad \sum_{dr}\sum_{a}\sum_{p}\sum_{intv} C_p \, EF_{a,dr} \, X_{a,dr,p}^{intv}$$

This objective function maximizes the weighted sum of efficiency factors. This motivates the model to carry cargo by using more efficient route-aircraft pairs. The definition and derivation of "route efficiency factor" is discussed in Chapter III. The constant C_p in the objective function motivates early deliveries. As the number of aircraft in early passes increases, the objective function value increases.

2. MOG Constraint

$$\sum_{a}\sum_{dr}\sum_{p \le t} CONSUMEP_{a,af,dr,t-p+1} \cdot X_{a,dr,p}^{intv} \le 1 \quad \forall\, t, af, intv$$

The MOG constraint ensures that, for a given time, the total amount of MOG consumed at an airfield does not exceed its limits. As described before p represents the pass start times. Selection of step size for the time interval and pass interval involves a trade-off between computational tractability and model fidelity. We choose 2 hours for the time and pass intervals as a compromise. In Figure 9, an example to explain the MOG constraint is constructed. In this figure, an aircraft leaves APOE at time 0.

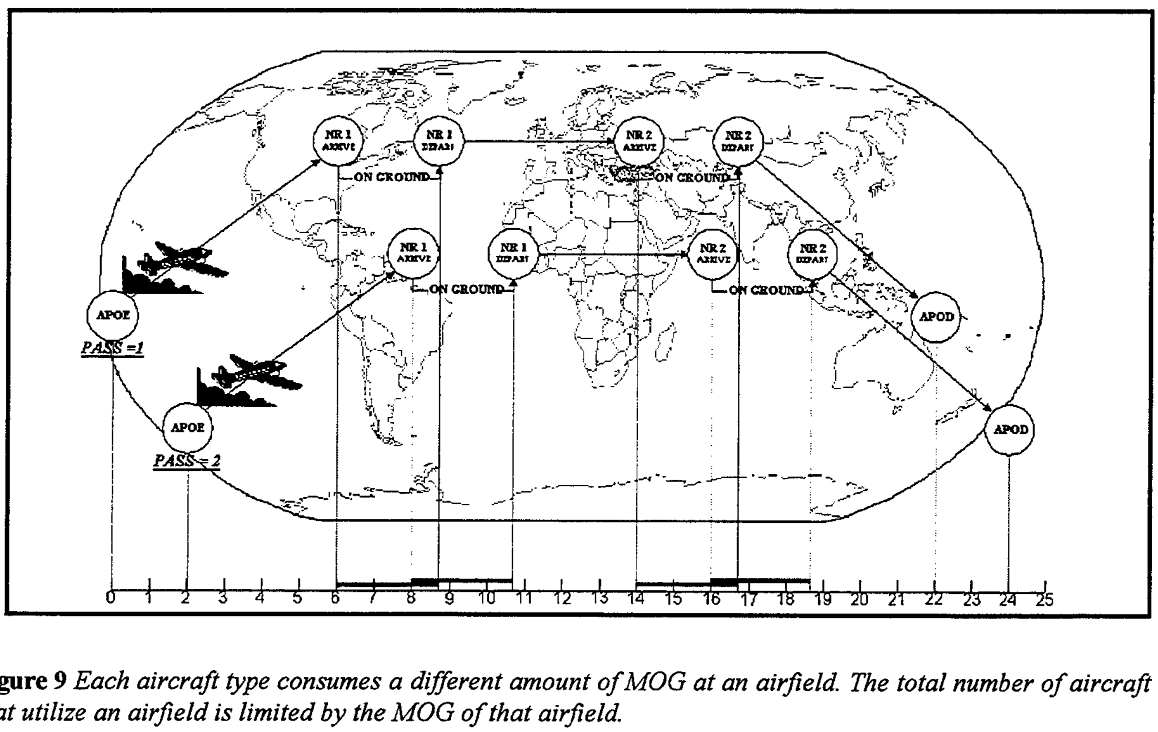

Figure 9 *Each aircraft type consumes a different amount of MOG at an airfield. The total number of aircraft that utilize an airfield is limited by the MOG of that airfield.*

Two hours later, the same APOE can launch another aircraft. This second launch constitutes the second pass. For the depicted route, NR01 is the first enroute base, and ground time at this airfield is 2.75 hours. An aircraft launched in the first pass arrives enroute base at time 6, and it stays on the ground until time 8.75. Aircraft in the second pass arrive at the first enroute base at time 8, and stay on the ground until time 10.75. For these two passes, MOG at NR01 airfield will be consumed throughout the time interval 6-10.75. At other times, this airfield is completely available for other aircraft.

Prior to running models, we need to make a decision how many passes we will have in the model. Considerations to define the number of passes are number of aircraft in the scenario and travel time of an aircraft from its origin to destination. We will continue launching aircraft until we run out of aircraft in the origin or the round trip time for the first set of aircraft elapses. The delivery model assumes that an aircraft leaves the system when it arrives at the destination airfield, but in reality the aircraft flies back to the origin airfield as soon as it completes off-loading. We will use this fact to define the number of passes we need. The set of aircraft we launch in the first pass are the most efficient aircraft. Thus, as soon as they arrive at the origin airfield we want to launch them again. Depending on the size of the aircraft fleet, we may run out of aircraft before the first set of aircraft arrive at the origin. The smaller of the following two conditions will determine the last pass:

1. The last pass prior to return of aircraft launched in pass 1

2. The last pass where the number of aircraft at the origin airfield is greater than 0.

As mentioned before, the parameter CONSUMEP is calculated only once. We shift the consumption of MOG delineated by this parameter in order to make it useful for all passes. The shifting conversion we use in the objective function assumes that time units for both p and t are the same. We will construct an example to explain the conversion in the CONSUMEP parameter.

We will use two hour intervals for both t and p.

$$t = p = 1, 2, 3, \ldots$$

This definition implies that the real time interval between consecutive t and p values is two hours. The CONSUMEP parameter is calculated for $p = 1$ and for all t's. If we are calculating the MOG consumption rate at an airfield af at $t = 10$, we need to sum the CONSUMEP parameter for all aircraft that use the airfield at $t = 10$. For an aircraft launched in the first pass ($p = 1$), MOG consumption is given by the parameter $CONSUMEP_{a,af,dr,10}$. Likewise, for an aircraft launched at $p = 2$, MOG must be "charged" at af in $t = 11$. Since $CONSUMEP_{a,af,dr,10}$ is the correct MOG amount to charge af for $p = 2$ missions, the correct indexing on CONSUMEP in the $t = 11$ MOG constraint is $t\text{-}p\text{+}1$, or 11-2+1=10.

3. Cargo Constraint

$$\sum_{a}\sum_{dr}\sum_{p} PAYLOAD_{a,dr} \cdot X_{a,dr,p}^{\text{intv}} \le TOTMOVE_{i}^{\text{intv}} \;\; \forall\; i, intv$$

This constraint ensures that total amount of cargo moved from a given origin airfield in a given interval is not more than cargo ready at that origin airfield.

4. Constraint for the number of Aircraft in the System

$$\sum_{dr}\sum_{p} X_{a,dr,p}^{intv} \le ACAVAIL_{a}^{intv} \quad \forall\; a, intv$$

The total number of missions we fly in each interval cannot be more than the aircraft fleet size of a given aircraft type in the interval, since each aircraft can do at most one mission in this simplified model.

Intervals we defined are completely independent, thus each interval gives a separable LP. Solving the Linear Programming model described above for each interval with different aircraft fleet structures, cargo amounts, and combinations of origin-destination pairs will stress the network in the extreme conditions. Thus, all the route-aircraft pairs that have a value greater than zero are candidates to be used by the *Throughput II*. We will retain all these routes and supply them to *Throughput II*.

B. RECOVERY ROUTES

We might have formulated a model considering both recoveries and deliveries simultaneously. The size of a model that includes recovery, delivery routes, and their associated decision variables would be very big, and take a prohibitively long time to run. Since we are trying to find the routes that are most likely to be used in *Throughput II,* the number of missions flown on these routes are not of as much importance as the fact that the routes were used at all. In other words, stressing the network in different conditions gives us the best route structure to obtain maximum throughput in the extreme conditions. Any route-aircraft pair that gets a positive value is a potential pair that will be used in *Throughput II*. All others (with 0 value) should not affect the throughput amount significantly, so eliminating these routes will not alter the solution value much.

We have defined a "route efficiency factor" for recovery routes, too. Using the results of models for delivery routes we can formulate a similar model for recoveries. In this model we use the same index sets and parameters, but instead of index *dr* (delivery routes) we will use *rr* for recovery routes. Also we will define new decision variables, ***Y*** and ***NoGo***. In the recovery route model, X will represent fixed values rather than representing a decision variable; we will call this fixed value X' to avoid any confusion.

PARAMETERS:

$X'^{intv}_{a,dr,p}$	Value calculated for decision variable X in the delivery model,
P	Penalty constant for an aircraft that couldn't be sent back.
R	Constant between 0 and 1 that defines how much of the MOG consumption in the delivery model will be taken into account in the recovery model.

DECISION VARIABLES :

$Y^{intv}_{a,rr,p}$	Number of a/c of type *a* sent on route *rr* in pass *p* on interval *intv*;
$NoGo^{intv}_{a,p}$	Number of a/c of type *a* that couldn't be sent back in pass *p*;

The model for recovery routes will be as follows :

1. Objective Function

$$\text{Maximize} \quad \sum_{rr}\sum_{a}\sum_{p}\sum_{intv} C_p \cdot EF_{a,rr} \cdot Y^{intv}_{a,rr,p} - \sum_{a}\sum_{p}\sum_{intv} P \cdot NoGo^{intv}_{a,p}$$

With this objective function we motivate aircraft to fly back. The objective function value will increase with the number of aircraft recovered. For recovery routes we

want to use the most efficient routes, so the route efficiency factor is a coefficient in the objective function. We also penalize the number of aircraft that couldn't fly back. ***P*** is penalty constant for decision variable ***NoGo,*** and it is at least as big as C_1.

2. MOG Constraint

$$\sum_{a}\sum_{rr}\sum_{p\leq t} CONSUMEP_{a,af,rr,t-p+1} \cdot Y_{a,rr,p}^{intv} \leq$$

$$1-\sum_{a}\sum_{dr}\sum_{p} R \cdot CONSUMEP_{a,af,dr,t-p+1} \cdot X'^{intv}_{a,dr,p} \quad \forall\, t, af, intv$$

Priority in the system should be for delivery routes, since the goal is to move as much cargo as possible. However, recovering the aircraft also has a great effect on the amount of cargo moved. Thus, in a model that handles recoveries and deliveries simultaneously, some of the MOG in busy airfields will be used by recovering aircraft. Unfortunately, the delivery model previously explained does not account for recovering aircraft. To be able to reflect this situation into our recovery model we modify the MOG amount consumed by delivery aircraft, multiplying by a constant *R* between 0 and 1. After making this modification for MOG consumed by X' s, the remaining MOG may be utilized by recovering aircraft in the recovery model. The MOG constraint in the recovery model reflects this availability.

3. Aircraft Balance Constraint

$$\sum_{rr} Y_{a,rr,p}^{intv} + NoGo_{a,p}^{intv} = \sum_{dr} \sum_{p' \ni p = p' + traveltime} X_{a,dr,p'}^{intv} + NoGo_{a,p-1}^{intv} \quad \forall\, k,a,p,intv$$

The aircraft balance constraint ensures that the total number of aircraft (of a particular type) launched from a destination airfield in a pass is less than or equal to the total number of aircraft offloading in that pass. The elastic variable *NoGo* keeps the problem feasible, when the available MOG is not enough to recover all required aircraft. We penalize this elastic variable in the objective function so that the model uses it as infrequently as possible.

C. SUMMARY

Simple mathematical models that stress the air mobility network may recommend route-aircraft pairs that can be used to reduce the size of *Throughput II*. The simplified models we developed in this chapter do not make viable recommendations for what missions to fly on their own, because we don't have explicit demand and required cargo delivery dates in our models. Basically, an air mobility problem is constrained by one or more of these three things:

1. Number of aircraft available in the system

2. Total amount of cargo ready to be moved from its origin airfield

3. MOG which represents aircraft handling capacity of an airfield

We formulated each of these as constraints in an LP. Mathematical models for delivery and recovery routes are constructed separately. We made some assumptions and simplifications in order to keep our models small. We used an air mobility scenario to test our mathematical models. The results of this work are presented in the next chapter.

V. RESULTS AND CONCLUSION

This chapter will verify the procedures we developed in Chapter III and Chapter IV. In those chapters we constructed procedures that will improve the run time of the mobility optimization model, while sacrificing little in solution quality. Retaining enroute airfields and their associated routes increases the throughput amount of the model, but the size of the model will increase and will take longer to run. We will observe changes in the size, optimality, and runtime of the mobility model when we apply the procedures described in this thesis. If we get a minor loss of optimality, while we cut the size of the model and improve the runtime, we will conclude that the procedures we introduced are successful.

To test our procedures, we will use a sample mobility scenario and we will compare the output of *Throughput II*, before and after the described procedures are applied. First we will give a brief description of the sample problem.

A. DESCRIPTION OF THE SAMPLE MOBILITY PROBLEM

The sample problem we used to test our procedures is called "Two MRC (Major Regional Contingency)," provided by AFSAA in August 1995. We try to move cargo

from origin airfields (APOE) to the destination airfields (APOD), using the assigned assets. The size of the data set is presented below :

Number of origin airfields (i)	:	4
Number of destination airfields (k)	:	2
Number of enroute airfields (e)	:	15
Number of routes (r)	:	123
Number of time periods (t)	:	47
Number of recovery route-aircraft pairs	:	126
Number of delivery route-aircraft pairs	:	150

The origin airfields are called OE04, OE05, OE06, OE07. The cumulative cargo amounts at the origin airfields are depicted in Figure 10. APOD airfields are called OD22, and OD27.

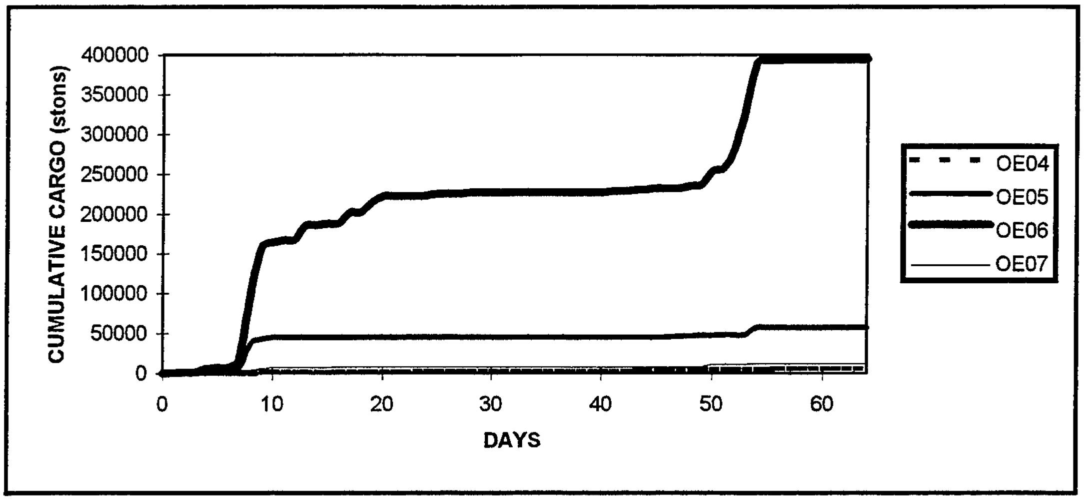

Figure 10 *Cumulative cargo amounts in stons at the origin airfields of the sample mobility problem*

Aircraft types, as well as their number and time introduced to the system, are shown in Table 1.

	C-5	C-17	C-141	KC-10	WBP	WBC	NBC
DAY 1	98	53	81	37	31	24	15
DAY 5	0	0	0	0	54	33	25
DAY 45	0	0	0	0	34	16	21

Table 1. *Types and numbers of aircraft introduced to the system in sample mobility problem.*

In the "Two MRC" scenario there are *123* routes in total. However not all the aircraft may fly on every route. The total number of feasible delivery route-aircraft pairs is

150 (about 21 routes per aircraft type on average), and total number of feasible recovery route-aircraft pairs is *126* (18 routes per aircraft type on the average).

B. TESTING PROCEDURE AND RESULTS

Step 1. We ran the problem with all the routes in *Throughput II*. It took *3* hours to generate and solve the problem. Model statistics that indicates the size of the generated model are as follows:

Single Equations : *198,237*

Single Variables : *210,727*

Non zero Elements : *1,217,745*

We observed that *Throughput II* used all of the available route-aircraft pairs in the scenario. The objective function value we obtained was *4847.5940*.

Summary of the mission numbers flown on routes are shown in Table 2.

MISSION NUMBERS FLOWN	LESS THAN 20	BETWEEN 20 AND 100	GREATHER THAN 100	**TOTAL**
NUMBER OF ROUTES	175	58	43	276

Table 2. *The distribution of missions to routes in Throughput II solution.*

Step 2. We ran the LP models described in the previous chapter for the following origin-destination combinations (each run constitutes a separate run).

1. **APOE** : OE04 ; **APOD** : OD22 ;
2. **APOE** : OE05 ; **APOD** : OD22;
3. **APOE** : OEO6 ; **APOD** : OD22;
4. **APOE** : OE07 ; **APOD** : OD22;
5. **APOE** : OE04 ; **APOD** : OD27;
6. **APOE** :OE05 ; **APOD** : OD27;
7. **APOE** : OE06 ; **APOD** : OD27;
8. **APOE** : OE07 ; **APOD** : OD27;
9. **APOE** : OE04, OE05, OE06, OE07 ; **APOD** : OD22;
10. **APOE** : OE04, OE05, OE06, OE07 ; **APOD** : OD27;
11. **APOE** : OE04, OE05, OE06, OE07 ; **APOD** : OD22, OD27;

The total runtime of all these models was *37* minutes. These runs suggested to eliminate *67* of the delivery route-aircraft pairs (which corresponds to *44.6* %), and *75* of the recovery route - aircraft pairs (which corresponds to *59.52* %). *112* of those routes to be eliminated had less than 20 missions, *19* had mission number between 20 and 100, and *11* had more than 100 missions.

Step 3. We removed those suggested route-aircraft pairs from the model and ran *Throughput II* again with new input. It took *45* minutes to generate and solve the model this time. This is a *75* % improvement in the run time. The total runtime of the route elimination model and *Throughput II* is *82* minutes. The improvement in the combined runtime is *55 %*. The objective function value we obtained was *4967.5750.* If we compare this figure with the objective function value we had, we sacrificed *2.475* % from optimality. Model statistics of the reduced model are as follows :

Single Equations	:	*92,891*
Single Variables	:	*100,505*
Non zero Elements	:	*572,176*

C. SUMMARY

The procedures introduced in this thesis were tested on a typical mobility problem. The route structure of a mobility problem is very important in terms of size and runtime of the model. We implemented the simple heuristic and its associated Linear Programming models described in Chapter IV in GAMS (General Algebraic Modeling System). We compared run time, optimality, and size of the models generated before and after the route elimination procedures were applied. The numerical results are summarized in Table 3.

MODEL	NUMBER OF DELIVERY ROUTES	NUMBER OF RECOVERY ROUTES	OBJECTIVE FUNCTION VALUE	RUNTIME (minutes)
ORIGINAL	150	126	4847.59	180
REDUCED	83	51	4967.57	45
% IMPROVEMENT	+ 44.67%	+ 59.53%	- 2.47%	+ 75%

Table 3. *This table shows the comparison of the results of the sample mobility problem. The row "ORIGINAL" indicates the values of the problem with complete input data. The row "REDUCED" shows the values we obtained after applying reduction procedures described in this thesis.*

Maintaining an entire set of aircraft-route pairs in a mobility problem returns a better objective function value. If one wants a quicker answer, then the size of the problem must be reduced. When the size of the original model is too large for contemporary solvers, route selection may be even more critical. A mobility network structure with fewer routes is also more realistic from an operations standpoint. We introduced techniques to compare route-aircraft pairs. The "Route Efficiency Factor" is the criterion

we used for our comparisons. Then, we formulated Linear Programs in order to cut the size of the problem and increase the runtime of the *Throughput II* with a small loss in the optimum value. Procedures described in this thesis can be implemented in the mobility model and a single model can be presented to the user. Thus, an easy to implement method to reduce the problem size of a large mobility linear program has been developed and tested as a result of this research.

APPENDIX A. THROUGHPUT II MODEL

The following is a brief summary of Throughput II (Morton *et. al.,* 1995).

A. INDICES

u	indexes units, e.g., 82nd Airborne
a	indexes aircraft types, e.g., C5, C141
t,t'	index time periods
b	indexes all airfields (origins, enroutes and destinations)
i	indexes origin airfields
k	indexes destination airfields
r	indexes routes

B. INDEX SETS

1. Airfield Index Sets

B	set of available airfields
$I \subseteq B$	origin airfields
$K \subseteq B$	destination airfields

2. Aircraft Index Sets

A	set of available aircraft types
$A_{bulk} \subseteq A$	aircraft capable of hauling bulk size cargo
$A_{over} \subseteq A_{bulk}$	aircraft capable of hauling over-sized cargo
$A_{out} \subseteq A_{over}$	aircraft capable of hauling out-sized cargo

Bulk cargo is palletized on 88 x 108 inch platforms, which can fit on any plane. Over-sized cargo is typically non-palletized rolling stock; it is larger than bulk cargo and can fit on a C141, C5 or C17. Out-sized cargo is very large non-palletized cargo that can fit into a C5 or C17 but not a C141.

3. Route Index Sets

R	set of available routes
$R_a \subseteq R$	permissible routes for aircraft a
$R_{ab} \subseteq R_a$	permissible routes for aircraft a that use airfield b
$R_{aik} \subseteq R_a$	permissible routes for aircraft a that have origin i and destination k
$DR_i \subseteq R$	delivery routes that originate from origin i
$RR_k \subseteq R$	recovery routes that originate from destination k

4. Time Index Set

T	set of time periods
$T_{uar} \subseteq T$	possible launch times of sorties for unit u using aircraft a and route r

The set T_{uar} covers the allowed time window for unit u, which starts on the unit's available-to-load date and ends on the unit's required delivery date, plus some extra time up to the maximum allowed lateness for the unit.

C. GIVEN DATA

1. Movement Requirements Data

$MovePAX_{uik}$	Troop movement requirement for unit u from origin i to destination k
$MoveUE_{uik}$	Equipment movement requirement in short tons (stons) for unit u from origin i to destination k
$ProBulk_u$	Proportion of unit u cargo that is bulk-sized
$ProOver_u$	Proportion of unit u cargo that is over-sized
$ProOut_u$	Proportion of unit u cargo that is out-sized

2. Penalty Data

$LatePenUE_u$	Lateness penalty (per ston per day) for unit u equipment

$LatePenPAX_u$ Lateness penalty (per soldier per day) for unit u troops

$NoGoPenUE_u$ Non-delivery penalty (per ston) for unit u equipment

$NoGoPenPAX_u$ Non-delivery penalty (per soldier) for unit u troops

$MaxLate$ Maximum allowed lateness (in days) for delivery

$Preserve_{at}$ Penalty (small artificial cost) for keeping aircraft a in mobility system at time t

3. Cargo Data

$UESqFt_u$ Average cargo floor space (in sq. ft.) per ston of unit u equipment

$PAXWt_u$ Average weight of a unit u soldier inclusive of personal equipment

4. Aircraft Data

$Supply_{at}$ Number of aircraft of type a that become available at time t

$MaxPAX_a$ Maximum troop carriage capacity of aircraft a

$PAXSqFt_{ua}$ Average cargo space (in sq. ft.) consumed by a unit u soldier fro aircraft a

$ACSqFt_a$ Cargo floor space (in sq. ft.) of aircraft a

$LoadEff_a$ Cargo space loading efficiency (<1) for aircraft a. This accounts for the fact that it is not possible in practice to fully utilize the cargo space.

$URate_a$ Established utilization rate (flying hours per aircraft per day) for aircraft a

5. Airfield Data

$MOGCap_b$ Aircraft capacity (in narrow-body equivalents) at airfield b

$MOGReq_{ab}$	Conversion factor to narrow-body equivalents for one aircraft of type a at airfield b
$MOGEff$	MOG efficiency factor (<1), to account for the fact that it is impossible to fully utilize available MOG capacity due to randomness of ground times

6. Aircraft Route Performance Data

$MaxLoad_{ar}$	Maximum payload (in stons) for aircraft a flying route r
$GTime_{abr}$	Aircraft ground time (due to onload or offload of cargo, refueling, maintenance, etc.) needed for aircraft a at airfield b on route r
$DTime_{abr}$	Cumulative time (flight time plus ground time) taken by aircraft a to reach airfield b along route r
$FltTime_{ar}$	Total flying hours consumed by aircraft a on route r
$CTime_{ar}$	Cumulative time (flight time plus ground time) taken by aircraft a on route r
$DaysLate_{uart}$	Number of days late unit u's requirement would be if delivered by aircraft a via route r with mission start time t

D. DECISION VARIABLES

1. Sortie Variables

X_{uart}	Number of aircraft a that airlift unit u via route r with mission start time during period t
Y_{art}	Number of aircraft a that recover from a destination airfield via route r with start time during period t

2. Aircraft Allocation and De-allocation Variables

$Allot_{ait}$	Number of aircraft a that become available at time t that are allocated to origin i
$Release_{ait}$	Number of aircraft a available at origin i in time t that are not scheduled for any flights from then on

3. Aircraft Inventory Variables

H_{ait}	Number of aircraft a inventoried at origin i at time t
HP_{akt}	Number of aircraft a inventoried at destination k at time t
$NPlanes_{at}$	Number of aircraft a in the air mobility system at time t

4. Airlift Quantity Variables

$TonsUE_{uart}$	Total stons of unit u equipment airlifted by aircraft a via route r with mission start time during period t
$TPAX_{uart}$	Total number of unit u troops airlifted by aircraft a via route r with mission start time during period t

5. Elastic (Nondelivery) Variables

$UENoGo_{uik}$	Total stons of unit u equipment with origin i and destination k that is not delivered in the prescribed time frame
$PAXNoGo_{uik}$	Number of unit u troops with origin i and destination k who are not delivered in the prescribed time frame

E. OBJECTIVE

minimize

$$\sum_{u}\sum_{a}\sum_{r\in R_a}\sum_{t\in T_{uar}} LatePenUE_u * DaysLate_{uart} * TonsUE_{uart} \tag{A.1}$$

$$+\sum_{u}\sum_{a}\sum_{r\in R_a}\sum_{t\in T_{uar}} LatePenPAX_u * DaysLate_{uart} * TPAX_{uart}$$

$$+\sum_{u}\sum_{i}\sum_{k}\left(NoGoPenUE_u * UENoGo_{uik} + NoGoPenPAX_u * PAXNoGo_{uik}\right)$$

$$+\sum_{a}\sum_{t} Preserve_{at} * NPlanes_{at}$$

The objective function minimizes the total weighted penalties incurred for late deliveries and non-deliveries. The model's secondary objective is to choose a feasible solution that maximizes unused aircraft.

F. CONSTRAINTS

$$\sum_{a\in A_{bulk}}\sum_{r\in R_{aik}}\sum_{t\in T_{uar}} TonsUE_{uart} + UENoGo_{uik} = MoveUE_{uik}, \tag{A.2}$$

$$\forall\ u,i,k\!: MoveUE_{uik} > 0$$

$$\sum_{a\in A_{out}}\sum_{r\in R_{aik}}\sum_{t\in T_{uar}} TonsUE_{uart} + UENoGo_{uik} \geq ProOut_u * MoveUE_{uik}, \tag{A.3}$$

$$\forall\ u,i,k\!: MoveUE_{uik} > 0 \quad .$$

$$\sum_{a \in A_{ovr}} \sum_{r \in R_{aik}} \sum_{t \in T_{uar}} TonsUE_{uart} + UENoGo_{uik} \geq \tag{A.4}$$

$$(ProOver_u + ProOut_u) * MoveUE_{uik}, \quad \forall\, u,i,k: MoveUE_{uik} > 0$$

$$\sum_{a} \sum_{r \in R_{aik}} \sum_{t \in T_{uar}} TPAX_{uart} + PaxNoGo_{uik} = MovePAX_{uik}, \tag{A.5}$$

$$\forall\, u,i,k: MovePAX_{uik} > 0$$

$$\sum_{r} \sum_{r \in DR_i} X_{uart} + H_{ait} + Release_{ait} = \tag{A.6}$$

$$H_{ai,t-1} + Allot_{ait} + \sum_{r \in R_{ai}} \sum_{t' + [CTime_{ar}] = t} Y_{art'}, \quad \forall\, a,i,t$$

$$\sum_{r \in RR_k} Y_{art} + HP_{akt} = HP_{ak,t-1} + \sum_{u} \sum_{r \in R_{ak}} \sum_{\substack{t' \in T_{uar} \\ t' + [CTime_{ar}] = t}} X_{uart'}, \qquad \forall\, a,k,t \tag{A.7}$$

$$\sum_{t'=1}^{t} \sum_{i} Allot_{ait} \leq \sum_{t'=1}^{t} Supply_{at}, \qquad \forall\, a,t \tag{A.8}$$

$$NPlanes_{at} = \sum_{t'=1}^{t} \sum_{i} Allot_{ait'} - \sum_{t'=1}^{t} \sum_{i} Release_{ait'}, \qquad \forall\, a,t \tag{A.9}$$

$$\sum_{r\in R_a}\sum_{t'=1}^{t}\sum_{u} K_{artt'} * X_{uart'} + \sum_{r\in R_a}\sum_{t'=1}^{t} K_{artt'} * Y_{art'} + \sum_{i}\sum_{t'=1}^{t} H_{ait'} \tag{A.10}$$

$$+\sum_{k}\sum_{t'=1}^{t} HP_{akt'} \leq \sum_{t'=1}^{t} NPlanes_{at}, \qquad \forall\, a,t$$

where

$$K_{artt'} = \begin{cases} t - t' + 1, & \text{if} \quad t' \leq t < t' + CTime_{ar} - 1 \\ CTime_{ar}, & \text{if} \quad t \geq t' + CTime_{ar} - 1 \end{cases}$$

$$TPAX_{uart} \leq MaxPAX_{a} * X_{uart}, \qquad \forall\, u,a,r,t\colon t \in T_{uar} \tag{A.11}$$

$$TonsUE_{uart} + PAXWt * TPAX_{uart} \leq MaxLoad_{ar} * X_{uart}, \tag{A.12}$$

$$\forall\, u,a,r,t\colon t \in T_{uar}$$

$$PAXSqFt_{a} * TPAX_{uart} + UESqFt_{u} * TonsUE_{uart} \leq \tag{A.13}$$

$$ACSqFt_{a} * LoadEff_{a} * X_{uart}, \qquad \forall\, u,a,r,t\colon t \in T_{uar}$$

$$\sum_{u}\sum_{r\in R_a}\sum_{t\in T_{uar}} FltTime_{ar} * X_{uart} + \sum_{r\in R_a}\sum_{t} FltTime_{ar} * Y_{art} \leq \tag{A.14}$$

$$\sum_{t} URate_{a} * NPlanes_{at}, \qquad \forall\, a$$

$$\sum_{u}\sum_{a}\sum_{r\in R_a}\sum_{\substack{t'\in T_{uar}\\ t'+[DTime_{abr}]=t}}\left(MOGReq_{ab}*GTime_{abr}/24\right)*X_{uart'} \tag{A.15}$$

$$+\sum_{a}\sum_{r\in R_a}\sum_{t'+[DTime_{abr}]=t}\left(MOGReq_{ab}*GTime_{abr}/24\right)*Y_{art'}$$

$$\leq MOGEff*MOGCap_b, \qquad \forall\, b,t$$

A.2 Demand satisfaction constraints for all classes of cargo
A.3 Demand satisfaction constraints for out-sized cargo
A.4 Demand satisfaction constraint for over-sized cargo
A.5 Demand satisfaction for troops
A.6 Aircraft balance constraints at origin airfields
A.7 Aircraft balance constraints at destination airfields
A.8 Aircraft balance constraints for allocations to origins
A.9 Aircraft balance constraints accounting for allocations and releases
A.10 Cumulative aircraft balance constraints
A.11 Troop carriage capacity constraints
A.12 Maximum payload constraints
A.13 Cargo floor space constraints
A.14 Aircraft utilization constraints
A.15 Airfield MOG constraints

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